United Kingdom
Mathematics Trust

Junior Mathematical Challenge

Solutions 2023

For reasons of space, these solutions are necessarily brief.

There are more in-depth, extended solutions available on the UKMT website, which include some exercises for further investigation:

www.ukmt.org.uk

1. **C** $3202 - 2023 = 1179$.

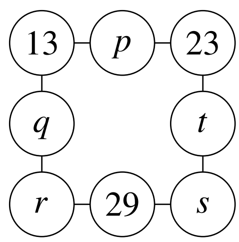
2. **D** Of the options given, only 4 is not a factor of 30. So four of the options are factors of 30.

3. **B** $\dfrac{1+2+3+4+5}{6+7+8+9+10} = \dfrac{15}{40} = \dfrac{3}{8}$.

4. **C** Of the options given, 12 is divisible by 2, 24 is divisible by 8 and 36 is divisible by 18. However, 72 is not divisible by 14 and 96 is not divisible by 54. Therefore, since we are told that one of these is the largest two-digit number that is divisible by the product of its digits, 36 is that number.

5. **E** Jumpy's approximate average speed in m/s was $\dfrac{100}{20} = 5$.

6. **D** As the sum of each row is 43, $p = 43 - (13 + 23) = 7$.
Also $r + s = 43 - 29 = 14$. The missing primes are all different so r and s are 3 and 11 in some order. If $r = 3$, then $q = 43 - (13 + 3) = 27$, which is not prime. However, if $r = 11$ then $q = 43 - (13 + 11) = 19$, which is prime. So $r = 11$. The square can now be completed: $s = 3$ and $t = 43 - (23 + 3) = 17$.
Therefore the sum of the five missing primes is $7 + 19 + 11 + 3 + 17 = 57$.



7. **B** The largest two-digit multiple of 2 is 98, while the smallest three-digit multiple of 3 is 102. Their difference is $102 - 98 = 4$.

8. **D** $0^2 + 1^2 = 0 + 1 = 1$; $1^2 + 2^2 = 1 + 4 = 5$; $2^2 + 3^2 = 4 + 9 = 13$; $3^2 + 4^2 = 9 + 16 = 25$; $4^2 + 5^2 = 16 + 25 = 41$; $5^2 + 6^2 = 25 + 36 = 61$. Of the six sums, 5, 13, 41 and 61 are prime.

9. **A** As $LM = LN$, $\angle LNM = \angle LMN$. Therefore, $3x - 20 = 2x + 8$. So $x = 28$.
Hence $\angle LNM = \angle LMN = (3 \times 28 - 20)^\circ = 64^\circ$.
The sum of the interior angles of a triangle is 180°, so $4y - 8 = 180 - 2 \times 64 = 180 - 128 = 52$.
Hence $y = (52 + 8) \div 4 = 15$.

10. **A** Let the length of the right hand edge of the shape be h cm. Then $2(h + 18) = 60$.
Therefore $h = 12$.
So the area, in cm^2, of the shape is $18 \times 12 - 6 \times (12 - 8) = 216 - 6 \times 4 = 216 - 24 = 192$.

11. C As 120 is a multiple of 4, Scrooge is able to use all of the tea bags when he uses two dried tea bags per cup and when he uses four dried tea bags per cup.
Hence the required number of 'decent' cups of tea is $120+120\div 2+120\div 4 = 120+60+30 = 210$.

12. E In the 20 minutes between 1:50 pm and 2:10 pm, Brian slithered $(210 - 150)$ cm = 60 cm.
So Brian slithered at a rate of 3 cm per minute and therefore took 50 minutes to slither 150 cm.
Hence Brian started his slither at 1 pm.

13. D The outer square has area $64\,\text{cm}^2$, so its side-length is 8 cm. Therefore the sum of the length and the breadth of one of the four congruent rectangles is 8 cm. Hence the perimeter of one of these rectangles is 2×8 cm = 16 cm.
(*Note that the perimeter of one of the rectangles depends only on the area of the outer square.*)

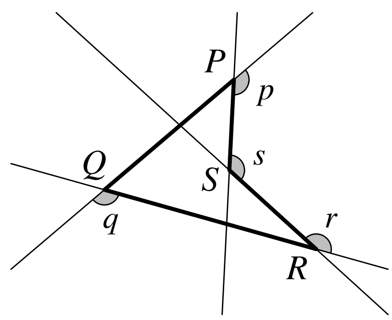
14. E Looking at the units column, we note that $3x = 7, 17$ or 27. However, of these, only $3x = 27$ has an integer solution, so $x = 9$ and there is a carry of 2 into the tens column.
Looking at the tens column, we now note that $2 + 7 + 2y = 9, 19$ or 29. Hence $y = 0, 5$ or 10, but 10 is not a single digit.
If $y = 0$, then there is no carry from the tens column to the hundreds column, so we have $7+6+0$ in the hundreds column which gives a total of 13, not the 19 we require.
However, if $y = 5$, then there is a carry of 1 from the tens column to the hundreds column so we have $1 + 7 + 6 + 5$ in the hundreds column giving a total of 19, which is correct. So $y = 5$.
Therefore $x + y = 9 + 5 = 14$.

15. B The scheduled leaving time of 17:48 is 12 minutes before 18:00, so the journey time, in minutes, should have been $12 + 25 = 37$. So the actual journey time, in minutes, was $2 \times 37 = 74$, that is 1 hour 14 minutes. Now the train left 4 minutes late, so the leaving time was 17:52.
Therefore the arrival time was 1 hour 14 minutes later than 17:52, that is 19:06.

16. D As the first two digits and the last two digits make up two two-digit numbers that are both multiples of 11, for example 33 and 55, the sum of the digits of the PIN is even. Note that the largest possible digit sum of a four-digit number is 36. Therefore, since the sum of the digits of the PIN is an even multiple of 11, that sum must be 22.
So there are eight possibilities for Amrita's PIN: 2299, 3388, 4477, 5566, 6655, 7744, 8833, 9922.

17. E We are given that $0 < p < q < 1$.
Therefore $q - p < q < 1$; $\quad p - q < 0$; $\quad \dfrac{p+q}{2} < \dfrac{1+1}{2} = 1$; $\quad \dfrac{p}{q} < 1$; $\quad \dfrac{q}{p} > 1$.
So $\dfrac{q}{p}$ is the only one of the five expressions which is greater than 1 and hence is the largest.

18. A Let the marked angles, in degrees, be p, q, r, s, as shown.
Then the interior angles of quadrilateral $PQRS$ are, in degrees, $180 - p, 180 - q, 180 - r, 360 - s$.
The sum of the interior angles of a quadrilateral is 360°.
Therefore $180 - p + 180 - q + 180 - r + 360 - s = 360$.
Hence $p + q + r + s = 3 \times 180 = 540$.
So the sum of the marked angles is 540°.



19. **D** If Rangers did not score in the first half, then there were five possible half-time scores from 0 – 0 to 0 – 4. However, if Rangers scored one goal in the first half then there were four possible half-time scores from 1 – 1 to 1 – 4. Similar reasoning shows that there are three possible half-time scores if Rangers scored two goals in the first half, two possible half-time scores if Rangers scored three goals in the first half and one possible half-time score if Rangers scored four goals in the first half. It is also possible that all goals were scored in the first half and that the half-time score was 5 – 4.
So the possible number of half-time scores is $5 + 4 + 3 + 2 + 1 + 1 = 16$.

20. **A** Note that 1 across is a two-digit cube and therefore is 27 or 64. Also, 2 across is a two-digit square and hence is 16, 25, 36, 49, 64 or 81. Therefore 1 down must end with 1, 2, 3, 4, 6 or 8. If 1 across is 27, then the possibilities for 1 down are 21, 22, 23, 24, 26 and 28. Of these only 23 is prime. Alternatively, if 1 across is 64, then the possibilities for 1 down are 61, 62, 63, 64, 66 and 68. Of these only 61 is prime. So the two possibilities are that 1 across is 27, 1 down is 23 and 2 across is 36 or that 1 across is 64, 1 down is 61 and 2 across is 16. So the sum of the four digits in the crossnumber is $2 + 7 + 3 + 6 = 18$ or $6 + 4 + 1 + 6 = 17$. Of these two possibilities, only 17 is one of the options.

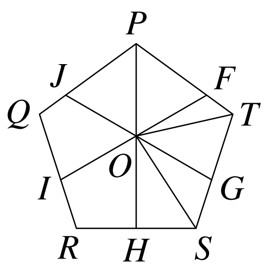
21. **B** Let the numbers of grey, pygmy, pink and white elephants be g, y, p and w respectively.
Then $g = 2y \ldots [1]$, $w = 3g \ldots [2]$, $p = 4w \ldots [3]$ and $w = y + 20 \ldots [4]$.
From [1] and [2] we get $w = 3 \times 2y = 6y$. Substituting for w in [4] gives $6y = y + 20$. So $y = 4$.
Therefore, $w = 6 \times 4 = 24$, $g = w \div 3 = 24 \div 3 = 8$ and $p = 4w = 4 \times 24 = 96$.
Hence the number of elephants in Eleanor's Emporium is $g + y + p + w = 8 + 4 + 96 + 24 = 132$.

22. **B** The second row is the only row whose product is a multiple of 5. The same is true of the second column. Hence e must be 5. Similarly, since 105 and 56 are the only multiples of 7 involved, then d must be 7. Hence, since $105 = 3 \times 5 \times 7$, it follows that $f = 3$.
Then column 3 shows that $3 \times c \times i = 36$ and so $c \times i = 12$. Since 3 has already been used for f, the only possible values for c and i are 2 and 6 in some order. If $c = 6$ then $a \times b = 18 \div 6 = 3$ and so one of a and b is 3, which isn't possible. Therefore $c = 2$ and then $i = 6$.

a	b	c	18
d	e	f	105
g	h	i	192
56	180	36	

(*It is left as an exercise for the reader to complete the grid and to confirm that it does include all the positive integers from 1 to 9 inclusive.*)

23. **E** As the six angles at O are equal, each is $360° \div 6 = 60°$. The five angles at the centre of the regular pentagon formed by joining each of its vertices to O are also all equal. So $\angle POT$ is $360° \div 5 = 72°$.
Therefore $\angle TOG = \angle POG - \angle POT = (2 \times 60 - 72)° = 48°$.
Consider triangle TOS : $\angle TOS = 72°$ and, as O is the centre of the pentagon, $SO = TO$. Hence $\angle OTS = \angle OST = (180 - 72)° \div 2 = 54°$.
In triangle TOG, $\angle TOG = 48°$ and $\angle OTG = \angle OTS = 54°$.
So $\angle TGO = (180 - 48 - 54)° = 78°$.



24. **E** Let the year this century in which Beatrix was born be '20xy'. Then her age on her birthday this year was $23 - 10x - y$. Therefore $2 + 0 + x + y = 23 - 10x - y$. Hence $11x + 2y = 21$.
If $x = 0$, then y is not an integer; if $x = 1$, then $y = 5$ and if $x > 1$ then y is negative.
So Beatrix was born in 2015 and was 8 years old on her birthday this year.
Let the year in which Beatrix's age on her birthday will be twice the sum of the digits of that year be '20pq'. As Beatrix was born in 2015, her age in '20pq' will be $10p + q - 15$.
Therefore $2(2 + 0 + p + q) = 10p + q - 15$. So $8p - q = 19$.
If $p \leq 2$, then q is negative; if $p = 3$, then $q = 5$ and if $p \geq 4$ then $q > 9$.
So the required year is 2035. Beatrix will then be 20, which is $2 \times (2 + 0 + 3 + 5)$.

25. **B** Let the number of spoons which Granny's neighbour received be z. Since the neighbour's share completes the distribution of all of Granny's spoons, z is also the number of spoons which were left after the son's share was removed. Therefore $z = \frac{z}{3} + 8$.
Hence $\frac{2z}{3} = 8$, that is $z = 12$.
Let the number of spoons remaining after Granny's daughter had taken her share of spoons be y.
So Granny's son received $\left(\frac{y}{3} + 8\right)$ spoons and there were then 12 spoons remaining.
Hence $\frac{y}{3} + 8 = y - 12$. Therefore $\frac{2y}{3} = 20$, that is $y = 30$.
Let the number of spoons in Granny's collection be x.
Then Granny's daughter received $\left(\frac{x}{3} + 8\right)$ spoons and there were then 30 spoons remaining.
Therefore $\frac{x}{3} + 8 = x - 30$. Hence $\frac{2x}{3} = 38$, that is $x = 57$. So the required digit-sum is 12.